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EFFECT OF THE PROCESS OF IONIZATION ON THE STRUCTURE AND STATE BEHIND A SHOCK WAVE IN A MAGNETIC FIELD

P. L. Gusika and G. S. Sukhov

ABSTRACT. The authors perform a qualitative analysis of the structure of a plane magnetohydrodynamic shock wave, taking into account the process of ionization on the basis of one-dimensional equations in a single-fluid approximation. The results of numerical calculations of the flow parameters behind the shock are compared with data obtained without taking the process of ionization into account.

This article presents a qualitative analysis of the structure of a plane /45* magnetohydrodynamic shock wave carried out on the basis of one-dimensional equations in a single-fluid approximation with consideration of the ionization process. The results of numerical calculations of the flow parameters behind the shock are compared with data obtained without taking the process of ionization into account.

Magnetohydrodynamic shock waves in a plasma without consideration of the ionization process within the wave have been studied quite completely [1-4].

Numerical calculation of the parameters of the gas behind the shock wave in the absence of electrical and magnetic fields with consideration of the ionization process was performed by Resler, Lin, and Kantrovits [5].

We will analyze the structure of a plane shock wave and calculate the flow parameters behind the wave with consideration of the process of single thermal ionization for a monatomic gas in a transverse magnetic field. We will consider that the time of establishing thermodynamic equilibrium is much less than the characteristic time of the problem, and the temperatures and velocities of the directed motion of the electrons, ions, and neutral atoms across the front of the shock wave are identical. Thus, the examination of plasma motion is given in a single-fluid approximation without consideration of the Hall effect and slipping past of the ions.

1. Analysis of One-dimensional Equations with Consideration of the Ionization Process. We will write a set of basic equations for a plane magnetohydrodynamic shock wave $u \perp E \perp B$ of a partially ionized ideal gas in a single-fluid approximation with a coordinate system moving together with the shock front [6,7]

*Numbers given in the margin indicate pagination in original foreign text.

$$\left.\begin{aligned}
&\frac{d(\rho u)}{dx}=0, \quad \frac{d(\rho u^2)}{dx}+\frac{d(p+p_m)}{dx}=\frac{d\tau}{dx},\\
&\frac{d(\rho u h_0)}{dx}=-\frac{1}{\mu}\frac{dB}{dx}\left(uB-\nu_m\frac{dB}{dx}\right)+\frac{d(\tau u-q)}{dx},\\
&\frac{d(uB)}{dx}=\frac{d}{dx}\left(\nu_m\frac{dB}{dx}\right),\\
&p=(n_a+n_i+n_e)kT=(1+\alpha)\rho RT,
\end{aligned}\right\} \qquad (1.1)$$

where $h_0=\left(\frac{5}{2}\frac{p}{\rho}+\alpha\frac{kT_i}{m_a}+\frac{u^2}{2}\right)$ is stagnation enthalpy [5,7]; $\alpha=\frac{n_e}{n_a+n_e}=\frac{n_i}{n_a+n_i}$ is the degree of ionization; $T_i=\frac{e\phi_i}{k}$ is the ionization temperature; ϕ_i is the first ionization potential; m_a is the atomic mass; $p_m=\frac{B^2}{2\mu}$; $\tau=\eta\frac{du}{dx}$; $q=-\lambda\frac{dT}{dx}$; λ,η, and $\nu_m=(\mu\sigma)^{-1}$ are the coefficients of thermal conductivity, and dynamic and magnetic viscosity respectively; the other designations are conventional. /46

In this set of equations ionization of the gas is taken into account along with viscous and Joulean dissipation (in the energy equation stagnation enthalpy includes the ionization energy; in all equations the pressure represents the sum of pressures of the plasma components).

The aforementioned set of equations (1.1) is not closed, since the character of the dependence of the dissipative coefficients η, λ, and ν_m and the degree of ionization α on the plasma parameters is not represented. To close system (1.1) we will consider η and λ dependent only on temperature, whereas ν_m and α are dependent on temperature and pressure; in this case a change of α by virtue of the assumptions adopted above concerning the thermal equilibrium of the plasma should be determined from the Saha equation:

$$\frac{\alpha^2}{1-\alpha}=C\frac{T^{3/2}}{\rho}\exp\left(-\frac{T_i}{T}\right), \qquad (1.2)$$

where C is a constant known for a given gas dependent on the statistical weights, quantum states of the ions and atoms, and atomic mass.

Consideration of the Saha equation in the general system of equations describing gas motion in a shock wave substantially hampers finding analytical solutions of the problem if any other simplifying assumptions are made relative to dissipative processes and the character of change of the magnetic field in the transitional zone. So far no analytical solutions are given in the literature either for the state behind the shock wave or for the transitional zone in the shock wave.

Let us examine the characteristics of the set of equations (1.1), (1.2).

For this purpose, by algebraic transformations we will obtain the equation for the flow velocity in the form of the law of inversion of actions [8], taking dissipative processes as the "external" actions

$$(\tilde{M}_*^2-1)\frac{d\ln u}{dx}=\frac{1}{\gamma_*(1+\xi A_*^2)p}\frac{d\tau}{dx}-$$
$$-\frac{2+\alpha(1-\alpha)\left(\frac{5}{2}+T_i/T\right)}{\left\{3+\alpha(1-\alpha)\left[\frac{3}{2}+\left(\frac{3}{2}+T_i/T\right)^2\right]\right\}\gamma_*}\frac{1}{(1+\xi A_*^2)pu}\times \quad (1.3)$$
$$\times\left[\frac{d\tau}{dx}\frac{du}{dx}-\frac{dq}{dx}+\frac{\nu_m}{\mu}\left(\frac{dB}{dx}\right)^2\right].$$

The following designations are adopted in equation (1.3)

$$\gamma_*=\frac{5+\alpha(1-\alpha)\left(\frac{5}{2}+T_i/T\right)^2}{3+\alpha(1-\alpha)\left[\frac{3}{2}+\left(\frac{3}{2}+T_i/T\right)^2\right]}; \quad (1.4)$$

$$A_*^2=\frac{B^2}{\mu\gamma_*p}=\frac{V_A^2}{a_*^2}; \quad \xi=\frac{d\ln B}{d\ln\rho}=-\frac{d\ln B}{d\ln u}; \quad (1.5)$$

$$\tilde{M}_*^2=\frac{\rho u^2}{\gamma_* p+\xi\frac{B^2}{\mu}}=\frac{u^2}{\tilde{a}_*^2}; \quad (1.6)$$

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$$\tilde{a}_*^2=a_*^2+\xi V_A^2=\gamma_*\frac{p}{\rho}+\xi\frac{2p_m}{\rho}. \quad (1.7)$$

These quantities represent: γ_* is the isentropic index; $A_*\tilde{M}_*$ are numbers analogous to the Alfven and Mach numbers; $\tilde{a}_*$ is the critical velocity analogous to the speed of sound.

It is necessary to note that the value of γ_* in (1.4) coincides with that found in [9,10] and when $\alpha = 0$ or $\alpha = 1$, $\gamma_* = 5/3$, i.e., coincides with the isentropic index of a perfect monatomic gas. Expression (1.7) is taken as the critical velocity, since when $u = \tilde{a}_*$ ($\tilde{M}_* = 1$) for a continuous change of velocity the total effect on the flow should be equal to zero (see equation (1.3)). Non-observance of this condition leads, when $\tilde{M}_* = 1$, to a critical flow

$$\left|\frac{d\ln u}{dx}\right|\to\infty,\ \left|\frac{d\ln\rho}{dx}\right|\to\infty,\ \left|\frac{d\ln p}{dx}\right|\to\infty \text{ etc.}$$

The essential fact is that the process of ionization does not appear as an independent "external" action, and its consideration leads to a change of the critical velocity and of the magnitudes of friction, heat conduction, and Joulean dissipation.

When $\alpha = 0$ (this means $\sigma = 0$ also) equations (1.1) describe flow in an ordinary gasdynamic wave, and the critical velocity (1.7) coincides here with the usual gasdynamic velocity of sound $\tilde{a}_*^2 = \frac{5}{3}\frac{p}{\rho} = a^2$.

In another limiting case when $\alpha = 1$ and $\nu_m = 0$ equations (1.1) correspond to flow of an ideally conducting completely ionized gas. The critical velocity $a_* = \sqrt{\frac{5}{3}\frac{p}{\rho} + 2\frac{p_m}{\rho}} = \sqrt{a^2 + v_A^2}$, which corresponds to the effective magnetohydrodynamic velocity of sound [7,11].

When $\alpha \neq 0$ and $\sigma \neq 0$ the following condition should be fulfilled at the boundaries of the shock wave [12]: in front of the wave

$$\tilde{M}_{*1}^2 = \frac{\rho_1 u_1^2}{\gamma_{*1} p_1 + 2p_{m1}} \geqslant 1$$

and behind the wave

$$\tilde{M}_{*2}^2 = \frac{\rho_2 u_2^2}{\gamma_{*2} p_2 + 2p_{m2}} \leqslant 1 \qquad (1.8)$$

Thus, in the transition zone the coefficient ξ changes within $0 \leqslant \xi \leqslant 1$, taking maximal values at the boundaries ($\xi_1 = \xi_2 = 1$); the critical velocity changes in like manner.

The value of the velocity of sound a_* can change, depending upon α and T_i/T, within limits from $a_*^2 = a^2 = \frac{5}{3}\frac{p}{\rho}$ (adiabatic sound) to $a_*^2 = a_\tau^2 = \frac{p}{\rho}$ (isothermal sound), i.e., $1 \leqslant \gamma_* \leqslant \frac{5}{3}$.

The equations in the form of the law of inversions of actions can be written for any parameter of the flow and any quantity which is a combination of these parameters (Mach number, M_*, etc). An examination shows that common for all these /48 equations is the presence of a critical velocity determined by formula (1.7) and a critical number $\tilde{M}_*$ (1.6) corresponding to it. Of independent interest is the equation of change of the degree of ionization

$$(\tilde{M}_*^2 - 1)\frac{d\ln\alpha}{dx} = \frac{(1-\alpha^2)(M_*^2 - \xi A_*^2)}{(1+\xi A_*^2)\left[2+\alpha(1-\alpha)\left(\frac{5}{2}+T_i/T\right)\right]p} \times$$

$$\times \frac{d\tau}{dx} - \frac{(1-\alpha^2)\left[\left(\frac{5}{2}+T_i/T\right) - \gamma_*\left(\frac{3}{2}+T_i/T\right)(M_*^2 - \xi A_*^2)\right]}{(1+\xi A_*^2)\left[5+\alpha(1-\alpha)\left(\frac{5}{2}+T_i/T\right)^2\right]pu} \times \qquad (1.9)$$

$$\times\left[\frac{d\tau}{dx}\frac{du}{dx}-\frac{dq}{dx}+\frac{\nu_m}{\mu}\left(\frac{dB}{dx}\right)^2\right] \qquad (1.9)$$

from which follows that the presence of a magnetic field can lead, for example, to a decrease rather than an increase of ionization along the flow. The degree of ionization, just as the flow velocity, remains constant in the absence of dissipative processes ($\eta = \lambda = \nu_m = 0$).

2. Structure of a Shock Wave in Ionized Gas in a Transverse Magnetic Field. Integration of equations (1.1) with respect to x leads to the following set of equations:

$$\left.\begin{aligned}
&\rho u = m, \quad \eta\frac{du}{dx} = mu + p + \frac{B^2}{2\mu} - C_1,\\
&\lambda\frac{dT}{dx} = -\eta\frac{du}{dx}u + pu\left(\frac{5}{2}+\frac{\alpha}{1+\alpha}\frac{T_i}{T}\right)+\frac{mu^2}{2}-\frac{BE}{\mu}-C_2,\\
&\nu_m\frac{dB}{dx} = uB - E, \quad p = (1+\alpha)\rho RT,
\end{aligned}\right\} \qquad (2.1)$$

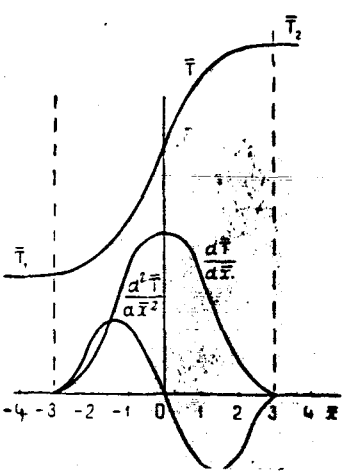

Figure 1. Illustration of the Effect of Heat Conduction on Shock-wave Structure.

where C_1 and C_2 are integration constants determined from the flow parameters at the boundary of the shock wave; m is the constant flow of mass; E is an electrical field constant along x. These equations cannot be reduced to two equations dependent only on u and B in order to analyze them in a phase diagram u-B, since the degree of ionization entering in them is determined by the Saha equation (1.2). As a consequence of this the method of isoclinic lines is unsuitable for the problem in question.

Let us analyze qualitatively the structure of a shock wave, using for this purpose the equation for velocity in the form of the law of inversion of actions (1.3). We will examine at first a shock wave in an inviscid

heat-conducting gas in the absence of an electromagnetic field. Equation (1.3) for this case is substantially simplified and takes the form /49

$$(M_*^2-1)\frac{d\ln u}{dx}=$$

$$=-\frac{2+\alpha(1-\alpha)\left(\frac{5}{2}+T_i/T\right)}{3+\alpha(1-\alpha)\left[\frac{3}{2}+\left(\frac{3}{2}+T_i/T\right)^2\right]}\frac{\lambda}{\gamma_* pu}\frac{d^2T}{dx^2}. \qquad (2.2)$$

This equation characterizes the change of velocity from the value in a homogeneous flow in front of the shock wave to its value behind the wave. In the supersonic part of the shock layer, where $M_* > 1$, the condition $\frac{d^2T}{dx^2} > 0$ must be fulfilled for continuous slowing down, as follows from (2.2) and since in the shock wave $\frac{dT}{dx} > 0$, the curve of the change of temperature in this layer has a concavity. This completely agrees with the numerical calculations of the shock-wave structure [1,3] and the experimental measurements (see, for example, Sherman's experiments [12]). Upon achieving $M_* = 1$, for a continuous transition from the supersonic layer to the subsonic layer the right-hand side of equation (2.2) should vanish, i.e., $\frac{d^2T}{dx^2} = 0$, which corresponds to the point of inflection of the curve of the temperature distribution in the shock wave. In the layer where $M_* < 1$, the curve of the change of temperature has a convexity.

If we are dealing with the motion of an inviscid and nonheat-conducting gas in a magnetic field, then, as we see from equation (1.3), in this case a continuous transition through the critical velocity within the shock front is impossible. The Joulean heat liberated in the flow leads to slowing down of the supersonic flow to a velocity $u_2 = a_{*2}$, or, as investigations have shown, $\tilde{a}_{*2} = a_{*2}$, since at this point a critical flow occurs and $\xi = 0$. Therefore, the presence of just Joulean dissipation does not lead to a continuous structure of the shock layer, the existence of which becomes possible when viscosity or heat conductivity is considered. Let us examine the characteristics of the structure of a shock wave in an electrical-conducting, heat-conducting, but inviscid gas. Assuming in (1.3) $\tau = 0$, we obtain:

$$(\tilde{M}_*^2-1)\frac{d\ln u}{dx}=$$

$$=-\frac{2+\alpha(1-\alpha)\left(\frac{5}{2}+T_i/T\right)}{3+\alpha(1-\alpha)\left[\frac{3}{2}+\left(\frac{3}{2}+T_i/T\right)^2\right]}\frac{\lambda\frac{d^2T}{dx^2}+\frac{\nu_m}{\mu}\left(\frac{dB}{dx}\right)^2}{(1+\xi\tilde{A}_*^2)\gamma_* pu}. \qquad (2.3)$$

On analyzing this equation we can conclude that in the super-critical portion of the shock front ($\tilde{M}_* > 1$) slowing down will occur if at first $\frac{d^2T}{dx^2} > 0$, then < 0 when $\left|\frac{d^2T}{dx^2}\right| < \frac{\nu_m}{\lambda\mu}\left(\frac{dB}{dx}\right)^2$, and in the sub-critical part ($\tilde{M}_* < 1$) when $\frac{d^2T}{dx^2} < 0$ for $\left|\frac{d^2T}{dx^2}\right| > \frac{\nu_m}{\lambda\mu}\left(\frac{dB}{dx}\right)^2$. A continuous transition through the critical velocity in the wave is possible if together with the advent of the equality $\tilde{M}_* = 1$, the equality $\frac{d^2T}{dx^2} = -\frac{\nu_m}{\lambda\mu}\left(\frac{dB}{dx}\right)$ or in dimensionless form /50

$$\frac{\lambda}{\lambda_1}\frac{d^2\bar{T}}{d\bar{x}^2} = -\frac{2\eta_1}{5}(1+\alpha_1)\frac{Pr_1}{Pr_{m1}}A_{*1}^2\frac{\nu_m}{\nu_{m1}}\left(\frac{d\bar{B}}{d\bar{x}}\right)^2, \tag{2.4}$$

where

$$Pr_1 = \frac{5\eta_1}{2\lambda_1}; \quad Pr_{m1} = \frac{\eta_1}{\nu_{m1}\rho_1}; \quad \bar{T} = \frac{T}{T_1}; \quad \bar{B} = \frac{B}{B_1} \text{ and } \bar{x} = \frac{x}{l}.$$

is also fulfilled.

The point of inflection of the curve of the change of temperature in the boundary layer lies in the supercritical region, and the transition through the critical region is observed in the area of heat removal caused by molecular transfer of heat against the gas flow when the Joulean heat being released is exactly equal to the heat removed from this layer (2.4). The fulfillment of this condition for gases in the temperature range $T \sim 10^3 - 10^5$°K and $B \sim 0.1$-10 Wb/m^2 requires that the temperature gradient be appreciably greater than the induction gradient of the magnetic field. This is arrived at by a numerical estimate of coefficients (2.4) for the selected values of T and B:

$$\frac{d^2\bar{T}}{d\bar{x}^2} \sim 10^6\left(\frac{d\bar{B}}{d\bar{x}}\right)^2. \tag{2.5}$$

A similar result was obtained for the first time by numerical calculation of the structure of a magnetohydrodynamic shock wave without consideration of the ionization process in work [1] and was explained later in works [2,3]. In weak magnetic fields the temperature and magnetic induction gradients can be commensurable or simply equal, and in the limit, in a very weak field, the dissipation associated with it can obviously be disregarded, and the continuous structure will be due to thermal conductivity.

Thus, the MHD wave in strong magnetic fields can conditionally be considered as consisting of two parts: a change of the magnetic field occurs in one part due to its interaction with the conducting flow, and an ordinary gasdynamic shock occurs in the other part in the presence of a constant magnetic field, which in [2] is called an "isomagnetic" shock. If the gas in front of the shock has a very low electrical conductivity, then from (2.3) and (2.4) follows the assumption that, at first there will be an isomagnetic shock and then a comparatively slow change of the flow parameters together with a change of

magnetic induction will be observed.

The character of the change in the degree of ionization in the front of the shock wave substantially depends upon the magnetic field strength. In an inviscid gas the equations of the law of inversion of actions for velocity (1.3) and degree of ionization (1.9) differ by coefficients: the sign of the coefficient of the right-hand side of (1.9) in the supercritical regime depends on the magnitude of magnetic induction, whereas in (1.3) it is always positive. Therefore, in the supercritical zone of the shock wave when $A_* < M_*$ we can observe a drop in the degree of ionization, whereas in the subcritical part it always increases.

In like manner we can analyze the structure of the wave in a viscous partially ionized gas. Without dwelling on a detailed examination, we will indicate only that the continuous transition through the critical velocity occurs, as follows from (1.3), with mutual compensation of the liberated Joulean heat and heat of friction by heat transfer into the adjacent layers and by work of the forces of internal friction which is performed in the gas. /51

3. State of Plasma Behind the Magnetohydrodynamic Shock Wave. To find the plasma parameters behind the shock wave in a magnetic field we must consider the flow to be homogeneous both before and after the shock, i.e., $du/dx = dT/dx = dB/dx = 0$. Then from equations (2.1) and (1.2) provided $\sigma_1 \neq 0$ we obtain

$$\left.\begin{aligned} &\rho_1 u_1 = \rho_2 u_2, \quad p_1 + \rho_1 u_1^2 + \frac{B_1^2}{2\mu} = p_2 + \rho_2 u_2^2 + \frac{B_2^2}{2\mu}, \\ &\frac{5}{2}\frac{p_1}{\rho_1} + \alpha_1 R T_i + \frac{u_1^2}{2} + \frac{B_1^2}{\mu\rho_1} = \frac{5}{2}\frac{p_2}{\rho_2} + \alpha_2 R T_i + \frac{u_2^2}{2} + \frac{B_2^2}{\mu\rho_2}, \\ &\frac{p_1}{(1+\alpha_1)\rho_1 T_1} = \frac{p_2}{(1+\alpha_2)\rho_2 T_2}, \quad u_1 B_1 = u_2 B_2, \end{aligned}\right\} \tag{3.1}$$

$$\frac{\alpha_1^2}{1-\alpha_1}\frac{\rho_1}{T_1^{3/2}}\exp\left(\frac{T_i}{T_1}\right) = \frac{\alpha_2^2}{1-\alpha_2}\frac{\rho_2}{T_2^{3/2}}\exp\left(\frac{T_i}{T_2}\right). \tag{3.2}$$

In view of the complexity of system (3.1), (3.2) it is proposed in monograph [7] to seek the solution by the method of successive approximations. However, a numerical calculation showed that graphic solution of the system is better. Changing to dimensionless variables, we reduce system (3.1), (3.2) to two equations of the form

$$\alpha_2 = \alpha_1 - \frac{1+\alpha_1}{2\theta}(1-\bar{u})[\varkappa(4\bar{u}-1) - \pi(5+\bar{u}^{-1}) - 5], \tag{3.3}$$

$$\alpha_2^2=\left(\frac{1+\alpha_1}{1+\alpha_2}\right)^{3/2}\left(\frac{1-\alpha_2}{1-\alpha_1}\right)\frac{\alpha_1^2\bar{u}[\varkappa\bar{u}(1-\bar{u})+\pi(\bar{u}-\bar{u}^{-1})+\bar{u}]^{3/2}}{\exp\left\{\theta\left[\frac{1+\alpha_2}{(1+\alpha_1)[\varkappa\bar{u}(1-\bar{u})+\pi(\bar{u}-\bar{u}^{-1})+\bar{u}]}-1\right]\right\}}, \tag{3.4}$$

where

$$\bar{u}=\frac{u_2}{u_1}, \quad \theta=\frac{T_i}{T_1}, \quad \varkappa=\frac{\rho_1 u_1^2}{p_1}=\gamma_{*1}M_{*1}^2, \quad \pi=\frac{p_{m1}}{p_1}.$$

The value of the degree of ionization and the velocity behind the shock is found from a graphic solution of equations (3.3), (3.4). The other parameters behind the shock are determined by relations (3.1). The state of the gas in front of the shock is given by four parameters: M_{*1}, the gasdynamic Mach number of the oncoming flow; θ, the dimensionless temperature of single ionization; π, the parameter of magnetic pressure; α_1 the initial degree of ionization.

When performing numerical calculations, θ was considered to be equal to 9.0 and 18.2, which corresponds, for example, to argon at an initial temperature of $2 \cdot 10^4$ and 10^4°K; π was varied within 0.83 and 83.3, which encompassed the range of possible values of magnetic induction B_1 from 0.1 to 10 Wb/m^2.

To investigate the influence of the initial degree of ionization on the parameters behind the shock as a function of the number M_{*1} we will consider θ = 18.2, and π = 0.83. The results of the calculation (Figure 2) show that consideration of the ionization process leads to a more severe compression in the shock, exceeding the classical limit $\frac{\gamma+1}{\gamma-1}$ = 4. Upon an increase of the Mach number M_{*1} the curves of $\bar{\rho} = \rho_2/\rho_1$ pass through the extreme and tend to the limit $(\gamma + 1)/(\gamma - 1)$. With an increase of α_1 the extreme value of compression changes. The anomalous shape of the curves $\bar{\rho}(M_{*1})$ is explained by the fact that a rise of temperature $\bar{T} = \frac{T_2}{T_1}$ as a function of M_{*1} outstrips the rise of pressure $\bar{p} = \frac{p_2}{p_1}$. Figure 3 shows the effect of parameter θ on the state behind the shock. An increase of θ, as follows from the figure, causes the same effect as a decrease of α_1. /53

The effect of the magnetic field is primarily felt because the shock appears at a Mach number M_{*1} determined from condition (1.8): $\tilde{M}_{*1}^2 = \frac{M_{*1}^2}{1+A_{*1}^2} \geqslant 1$.

Therefore, on all graphs for a given π there is a well defined minimal value of M_{*1} starting from which there is a shock. With an increase of π the minimal value of M_{*1} increases, and compression in the shock, temperature, and pressure

diminish (Figure 4). The disappearance of the anomalous shape of the curve $\bar{\rho}(M_{*1})$ upon an increase of π is important. With an increase of π there is also an increase of $\tilde{M}_{*2}$ and M_{*2}, wherein $\tilde{M}_{*2} \leq 1$ and M_{*2} will be greater than or less than 1 depending upon M_{*1}.

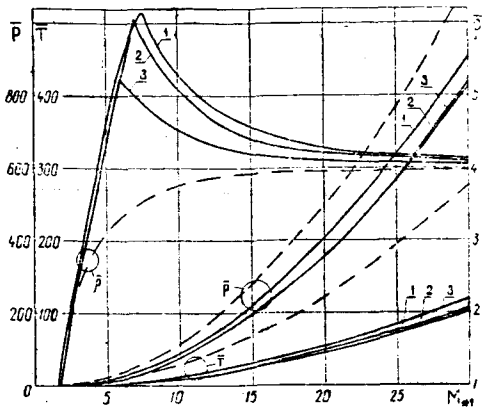

Figure 2. Change of pressure, temperature, and density behind shock as a function of the number M_{*1} of the oncoming flow ($\theta = 18.2$; $\pi = 0.83$).

——— 1) $\alpha_1 = 0.014$; 2) $\alpha_1 = 0.09$; 3) $\alpha_1 = 0.285$. --- calculation without consideration of ionization process.

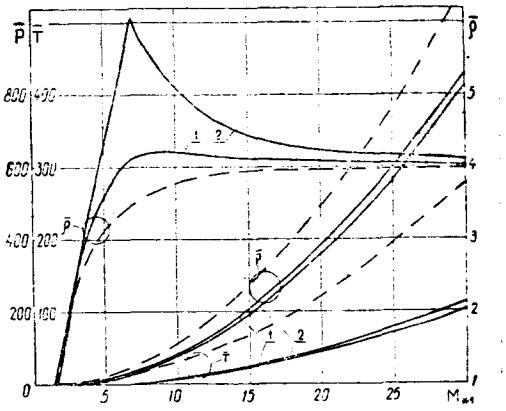

Figure 3. Change of flow parameters behind shock as a function of the number M_{*1} ($\alpha_1 = 0.09$; $\pi = 0.83$). ——— 1) $\theta = 9.0$; 2) $\theta = 18.2$. --- calculation without consideration of ionization process.

Summary

Consideration of the ionization process in magnetohydrodynamic flows with shock waves leads not only to quantitative differences but also to certain qualitatively new effects (change of the value of the critical velocity, anomalous shape of the curve $\bar{\rho}(M_{*1})$ and others).

The value of the critical velocity that was found depends on the degree of ionization, and when $Re_m \ll 1$ it coincides with the speed of sound in an ionized gas in the absence of a magnetic field [9,10].

The use of the law of inversion of actions permits a qualitative analysis of the structure of shock waves.

Calculation of plasma parameters behind a shock is performed with consideration of single ionization, which, strictly speaking, holds true for a gas whose atoms consist of a nucleus and one electron. Consideration of double ionization and ionization of a higher multiple would lead to an increase of the

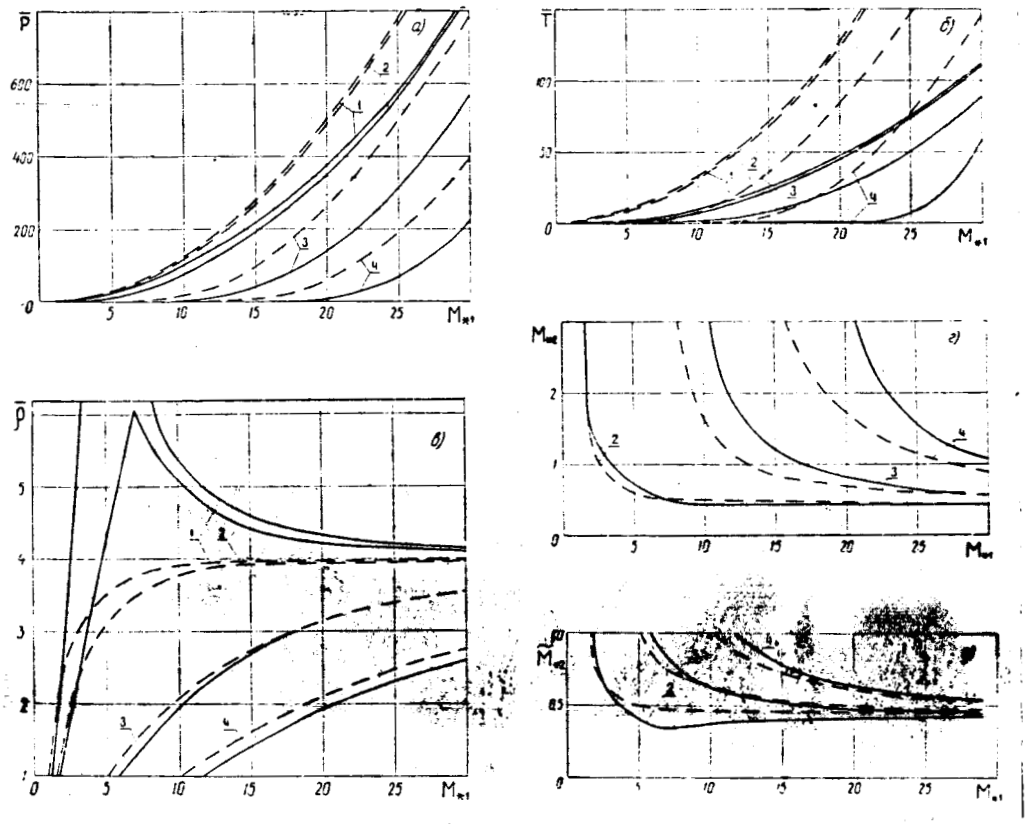

Figure 4. Effect of magnetic field on plasma parameters behind shock wave (α_1 = 0.09; θ = 18.2). ——— 1) π = = 0; 2) π = 0.83; 3) π = 21.0; 4) π = 83.3. --- calculation without consideration of ionization process.

extreme value of compression. Furthermore, it has been suggested that a shock wave arises and propagates in an ionized gas with an appreciable magnitude of electrical conductivity.

When shock waves propagate in a nonionized gas in the presence of a magnetic field, the interaction of the gas with the field occurs in the flow behind the shock wave or within the shock front where the conductivity of the gas $\sigma \neq 0$. In this case we can no longer write the condition on the shock $u_1B_1 = u_2B_2$. Calculation of a shock wave ionizing gas in a magnetic field can be performed by conditionally dividing the zone of the shock transition into two parts: one with zero conductivity and the other with infinite conductivity [13]. We must find $\varepsilon = \frac{u_2B_2}{u_1B_1}$ from an analysis of the structure of such a wave on the assumption that the gas at some temperature T* acquires an infinite electrical conductivity and begins to interact with the magnetic field. After this the parameters of the plasma behind the shock can be calculated by relations (3.1), (3.2) with consideration of ε.

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